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Space-Time Spaceless-Timeless Interactions

L. CORSIGLIA

Incol Foundation, Incorporated Post Office Box 3314, Merchandise Mart Station, Chicago, Illinois 60654, *and Department of Physics, University of California, Davis, California* 95616

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Abstract

An argument, based on Lorentz invariance for the number of discrete objects and Lorentz non-invariance for continuous physical quantities, is used to arrive at an uncertainty relation involving dipole moment and mass. Applied to a photon, a virtual dipole moment is defined and the photon itself is described as an electromagnetic wave. The small distance singularity in the Coulomb potential is removed by using a complex number for distance.

The number of discrete objects is invariant to a Lorentz transformation in special relativity theory (Eddington, 1924). One might then expect that the first order of infinity, the infinity of discreteness, would represent Lorentz invariant quantities. The second order of infinity, the infinity of continuity, could be used to represent quantities that are not Lorentz invariant, length and time for example (Møller, 1952). (Length and time are assumed not to be quantized in this paper. In fact, since $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ is the invariant space-time interval in special relativity (Møller, 1952), it may be that ds should be quantized and not dt or dx separately.) The mathematical existence of an order of infinity between the discrete and the continuous is not proven or disproven (Natanson, 1955). A possible physical meaning to an interface between discrete and continuous infinity can be postulated. The author has previously suggested that the uncertainty principle of quantum mechanics describes an interface or interaction between space-time and spacelessness-timelessness (Corsiglia, 1973). An interface between discrete and continuous infinity might also describe an interaction between space-time and spacelessness-timelessness. This paper attempts to find a way in which the orders of infinity can be at least heuristically important for physics.

The physical quantities charge, entropy (for reversible transformations), and action are invariants under a Lorentz transformation (Møller, 1952; Aharoni, 1965). Charge is measured in units of e, the charge of an electron. Entropy is represented in terms of $\ln(W)$, in which W is the number of micro-states corresponding to a macrostate (Sears, 1953). Action is measured in units of $\hbar$, Planck's constant (Feynman & Hibbs, 1965).

The physical quantities time, length, and mass are not invariants under a Lorentz transformation (Møller, 1952). Time, length, and mass will be taken as continuous quantities. The concept of mass as a continuous quantity can be supported by considering Terletskii's proof for the case in which the rest mass of a system of zero rest mass particles is greater than zero (Terletskii, 1968).

Heuristically, one might form the following correspondences: entropy and time, action and length, charge and mass. Continuing in a heuristic manner, how could one form uncertainty relations for these correspondences? We know already that $\Delta E \Delta t \geqslant \hbar$, in which ΔE is the uncertainty in energy and Δt is the uncertainty in time, and that $\Delta p \Delta x \geqslant \hbar$, in which Δp is the uncertainty in momentum and Δx is the uncertainty in position (Schiff, 1968). In each instance of forming an uncertainty relation, the discrete quantities of entropy and action were replaced by the not necessarily discrete quantities of energy and momentum. (Continuum states for energy and momentum exist in quantum mechanics (Schiff, 1968).) In forming an uncertainty relation from the charge-mass correspondence, discrete charge could be replaced by the electric dipole moment, P. Then, $\Delta P \Delta m \geqslant \hbar e/c$, in which c is the speed of light in a vacuum, provides an uncertainty relation between electric dipole moment and mass in terms of fundamental constants. In the case of current, a similar relation would hold for the magnetic dipole moment, M, in Gaussian units: $\Delta M \Delta m \geqslant \hbar e/c$.

Let us consider a possible physical interpretation of the uncertainty relation involving electric dipole moment and rest mass for the case of a photon. In the presence of a heavy mass (such as a nucleus), a photon of energy at least $2m_e c^2$ can split into an electron and a positron, each of mass m_e (Eisberg, 1961). Now, if we can consider the photon as some form of a dipole (not a perfect point dipole since $p = 0$ for the uncharged photon), say a virtual point dipole, then uncertainty in its moment would be related to uncertainty in its mass, a large enough mass uncertainty resulting in the formation of an electron-positron pair. Consider that, initially, the rest mass of the photon and the uncertainty of this rest mass are zero or practically zero (Goldhaber & Nieto, 1971). Consequently, the uncertainty in the virtual electric dipole moment, P', of the photon would approach infinity (P' is defined later). The object is then to find the form a photon dipole would take, a form which would describe a dipole of infinite uncertainty and yet produce an electromagnetic field.

If the photon is near a nucleus, then the virtual point dipole photon would be polarized in the Coulomb field of the nucleus. The uncertainty in the mass of the photon, which would be in the process of forming an electron-positron pair, would then increase. The uncertainty in the virtual moment of the photon should decrease from its previously mentioned infinite value according to our uncertainty relation. The newly separated plus and minus charge should form, at least temporarily, a dipole moment of finite value and finite uncertainty.

The above ideas can be further explored if we are willing to write a somewhat generalized expression for an electric dipole field, $\vec{E}$, as $\vec{E} = P\vec{f}(r, \theta, \phi)$, in which r, θ, ϕ represent spherical coordinates. The function f then gives the distribution in magnitude and direction of the electric dipole moment, P. A particular case of such an expression can be found in textbooks on electromagnetic theory (Reitz & Milford, 1960). Let us consider the electric dipole arrangement to be moving with a velocity v in the x-direction. In its rest frame, the electric dipole would present an electric field, $\vec{E}$, but not a magnetic field, $\vec{H}$. Further, let us Lorentz transform from the rest frame of the electric dipole to the frame of the observer, using the transformation velocity v. Hence, the observer, in the primed coordinate system, would measure, with $\gamma = (1 - v^2/c^2)^{-1/2}$, $\beta = v/c$, and $\vec{H} = 0$ (Aharoni, 1965):

$$\begin{aligned} E_{z'} &= \gamma E_z = \gamma P f_z & \qquad H_{z'} &= \gamma\beta P f_y \\ E_{y'} &= \gamma P f_y & H_{y'} &= -\gamma\beta P f_z \\ E_{x'} &= P f_x & H_{x'} &= H_x = 0 \end{aligned}$$

If we wish to apply this analysis to a photon, the following double limit must be taken: $v \to c$ and $P \to 0$. Then,

$$\begin{aligned} E_{z'} &= f_z \lim_{\substack{v\to c\\ P\to 0}} (\gamma P) & \qquad H_{z'} &= f_y \lim_{\substack{v\to c\\ P\to 0}} (\gamma\beta P) \\ E_{y'} &= f_y \lim_{\substack{v\to c\\ P\to 0}} (\gamma P) & H_{y'} &= -f_z \lim_{\substack{v\to c\\ P\to 0}} (\gamma\beta P) \\ E_{x'} &= f_x \lim_{\substack{v\to 0\\ P\to 0}} (P) = 0 & H_{x'} &= 0 \end{aligned}$$

But $\lim_{\substack{v\to c\\ P\to 0}} (\gamma P) \to 0/0$. This limit may exist just as $\lim_{x\to k} \{[f(x) - f(k)]/(x-k)\}$ can exist in calculus. Let us then define the virtual electric dipole moment of the photon as $P' = \lim_{\substack{v\to c\\ P\to 0}} (\gamma P)$. Then,

$$\begin{aligned} E_{z'} &= P' f_z & \qquad H_{z'} &= P' f_y \\ E_{y'} &= P' f_y & H_{y'} &= -P' f_z \\ E_{x'} &= 0 & H_{x'} &= 0 \end{aligned}$$

Also, in the primed system, $\vec{H}' = \vec{i}' x \vec{E}'$, in which $\vec{i}'$ is the unit vector in the x'-direction.

For a photon, our uncertainty relation must be written $\Delta P' \Delta m \geqslant \hbar e/c$.

P' is virtual because it has infinite uncertainty for $\Delta m = 0$ (photon), implying that P' cannot be measured for a photon, yet it leads to a picture of a photon as a transverse electromagnetic wave. Just as certain drawings can represent two aspects depending on the mental state of the observer (Perls *et al.*, 1951), electromagnetic radiation can present two aspects depending on the type of experiment used to measure the radiation. Wave-particle duality would not be an either–or situation, but the result of the interaction of a single radiation entity with instruments.

By determining $\hat{f}$ and P', theoretically one should be able to sum the individual photon fields to arrive at static, plane wave, and spherical wave electric fields for the conditions of a particular problem. Such summations, if possible, could lead to a better understanding of the differences between coherent and incoherent light.

It has been postulated that physically measurable quantities such as mass, length, and time consist not only of a measurable real part, but also of an immeasurable imaginary part (Gruber, 1972). Let us apply this postulate to the Coulomb potential between a charge $+e$ and a charge $-e$ separated by the complex distance $\hat{x} = x_R + ix_I$. Then,

$$\hat{V} = \frac{-e^2}{x_R + ix_I} = \frac{-e^2}{x_R + \dfrac{x_I^2}{x_R}} + i\frac{e^2}{x_I + \dfrac{x_R^2}{x_I}} = V_R - iV_I$$

An interesting aspect of this complex Coulomb potential is the following: $V_R \to 0$ as $x_R \to 0$ for x_I = constant, which will be assumed. The purpose here is simply to indicate how the imaginary (spaceless-timeless) part of a fundamental quantity such as length could influence a physical potential.

An imaginary potential represents a source or sink for particles (Schiff, 1968). If the plus charge is identified with a positron and the minus charge with an electron, then V_I could be identifiable, in the limit $x_R \to 0$, with the energy of annihilation, $2m_ec^2$. Then, as $x_R \to 0$, $e^2/x_I = -2m_ec^2$, or $x_I \approx -10^{-13}$ cm. And,

$$V_R = -\frac{e^2}{x_R + \dfrac{10^{-26}}{x_R}}$$

The real part of the Coulomb potential does not approach infinity as the charge separation, x_R, approaches zero. V_R remains attractive all the way to $x_R = 0$ and decreases for $x_R < 10^{-13}$ cm.

Charge is another fundamental quantity that could be regarded as complex. Then, $e^2 \to ee^* = e_R^2 + e_I^2$. The magnitude of the actual charge measured,

$|e_R|$, would then be less than the magnitude of the entire (bare) charge, $|e| = (e_R^2 + e_I^2)^{1/2}$. Also,

$$V_R = -\frac{e_R^2 + e_I^2}{x_R + \dfrac{10^{-26}}{x_R}}$$

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